Statically Indeterminate Problems and Problems Involving Two Materials

(Strength of Materials)

Dave Morgan
<dale.morgan@sait.ca>
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  - \( T_{BC} = 30 \text{ kN} \) (tension)
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- $T_{AB} = 20$ kN (tension)
Structures where forces can be determined using the static equilibrium equations alone ($\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M_A = 0$) are called \textit{statically determinate} structures. The previous example is a statically determinate structure.
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Statically indeterminate structures are often analysed using the conditions of axial deformation given by

$$\delta = \frac{PL}{AE}$$
Example: Consider a bar AB supported at both ends by fixed supports, with an axial force of 12 kN applied at C as illustrated. Find the reactions at the walls.
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**Solution:** First, draw a free body diagram:
**Example:** Consider a bar AB supported at both ends by fixed supports, with an axial force of 12 kN applied at C as illustrated. Find the reactions at the walls.

![Diagram of the bar AB with forces and dimensions]

**Solution:** First, draw a free body diagram:
\[ \Sigma F_x = R_A + R_B - 12 = 0 \]
Statically Indeterminate Problems

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\[ \Rightarrow R_A + R_B = 12 \]
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Statically Indeterminate Problems

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Statically Indeterminate Problems

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Two unknowns and a single equation; the problem is \textit{statically indeterminate}.

The supports at A and B are fixed so \( \delta_{AB} = 0 \).
\[ \Rightarrow \delta_{AC} + \delta_{CB} = 0 \]
\[ \Rightarrow \frac{-R_A \times 500}{AE} + \frac{R_B \times 400}{AE} = 0 \]
\[ \Rightarrow 400R_B = 500R_A \]

Now we have two equations and two unknowns; we can solve for \( R_A \) and \( R_B \).
Statically Indeterminate Problems

\[ R_A + R_B = 12 \]

\[ 400R_B = 500R_A \]
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\[ \Rightarrow R_A = 12 - R_B \]
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\[ \Rightarrow 900R_B = 6000 \]
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⇒ \[ R_A = 12 - R_B \]

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⇒ \[ 900R_B = 6000 \]

⇒ \[ R_B = 6.667 \text{kN} \]
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R_A + R_B &= 12 \\
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\end{align*} \]

\[\Rightarrow R_A = 12 - R_B\]

\[\Rightarrow 400R_B = 500(12 - R_B)\]

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\[\Rightarrow R_B = 6.667 \text{ kN}\]

\[\Rightarrow R_A = 5.333 \text{ kN}\]
Statically Indeterminate Problems

A 500 mm B

5.333 kN 12 kN C 6.667 kN
Exercise: Find $R_A$ and $R_B$ for the problem illustrated:
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**Solution:** Draw free body diagram
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**Statically Indeterminate Problems**

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Statically Indeterminate Problems

Solution: \[ R_A + R_B - P = 0 \]
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\[ \Rightarrow R_A = P - R_B \]
**Statically Indeterminate Problems**

![Diagram of a structure with forces](image)

**Solution:**

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\[ \delta_{AC} + \delta_{CB} = 0 \]

\[ \Rightarrow \frac{R_A \times a}{AE} + \frac{-R_B \times b}{AE} = 0 \]
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**Statically Indeterminate Problems**

![Diagram](image)

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\Rightarrow aP = (a + b)R_B
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\Rightarrow R_B = \left(\frac{a}{a+b}\right)P
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Statically Indeterminate Problems

Solution:

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\[ \Rightarrow a(P - R_B) = bR_B \]

\[ \Rightarrow aP = (a + b)R_B \]

\[ \Rightarrow R_B = \left( \frac{a}{a+b} \right) P \]

\[ \Rightarrow R_A = \left( \frac{b}{a+b} \right) P \]
Steel-reinforced concrete is used in the construction of many structures:

- Bridges
- Basements
- High-Rise Buildings
- Stadia, such as the SaddleDome or the Speed-Skating Oval
Concrete has a high load-bearing capacity in compression but is not very strong under a tensile load.
Problems Involving Two Materials

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- Steel rod has high load-bearing capacity in tension but buckles easily under compression.
Concrete has a high load-bearing capacity in compression but is not very strong under a tensile load.

Steel rod has high load-bearing capacity in tension but buckles easily under compression.

Combining steel rod and concrete gives a building material with both good tensile and compressive load-bearing qualities.
Steel in a concrete column also helps the concrete’s compressive strength:
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- When a column is loaded, it deforms \( \delta = \frac{PL}{AE} \)
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- When a column is loaded, it deforms \( \delta = \frac{PL}{AE} \)
- Under compression, \( \delta \) is negative and there is negative axial strain
- Consequently, there is a positive transverse strain \( \epsilon_t = -\mu \epsilon_a \)
- The concrete is under tension laterally
- Horizontal steel-reinforcing increases the lateral tensile strength of the column
A concrete footing is poured:

- It contains steel rebar throughout
- Steel extrudes from the top of the footing
- This will be attached to the steel for the column.
Steel is tied for the column
A frame is built around the steel and the concrete column is poured
Problems Involving Two Materials
We can use $\delta_C = \frac{P_C \cdot L_C}{A_C \cdot E_C}$ to calculate the deformation of concrete under a load.
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We can use $\delta_S = \frac{P_S \cdot L_S}{A_S \cdot E_S}$ to calculate the deformation of steel under a load.
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How can we calculate the deformation of a steel-reinforced concrete column?
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How can we calculate the deformation of a steel-reinforced concrete column?

- $E_C$ is not the same as $E_S$ so we cannot simply apply $\delta = \frac{P L}{A E}$ for the whole column.
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How can we calculate the deformation of a steel-reinforced concrete column?

- \( E_C \) is not the same as \( E_S \) so we cannot simply apply \( \delta = \frac{P_L}{A_E} \) for the whole column.

- We cannot solve this problem directly using the equations of statics, so this is a statically-indeterminate problem.
Example: A concrete column has a diameter of 300 mm. The column has 6 steel reinforcing rods. Each rod has a cross-sectional area of 200 mm$^2$. (See plan view)

$E_S = 210$ GPa and $E_C = 25$ GPa

The column is 1.15 m long and has a load of 1.37 MN is applied to a rigid steel plate at the top of the column (the plate distributes the load evenly over the top of the column).
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The column is 1.15 m long and has a load of 1.37 MN is applied to a rigid steel plate at the top of the column (the plate distributes the load evenly over the top of the column).

Find the stress in the steel and in the concrete, and the deformation under the load.
Solution: Let $P_S$ be the total reaction force of the six steel rods and $P_C$ the reaction force of the concrete.
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\[ P_S + P_C = 1370 \text{ kN} \]
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We have a single equation with two unknowns, so the problem is statically indeterminate.
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We have a single equation with two unknowns, so the problem is statically indeterminate. The concrete and the steel rods deform (contract) by the same amount, $\delta$, so...

\[ \frac{P_S \cdot L_S}{A_S \cdot E_S} = \delta = \frac{P_C \cdot L_C}{A_C \cdot E_C} \]
**Problems Involving Two Materials**

**Solution:**

\[
\frac{P_S \cdot L_S}{A_S \cdot E_S} = \frac{P_C \cdot L_C}{A_C \cdot E_C}
\]

\[1370 \text{ kN}\]
Problems Involving Two Materials

Solution:

\[
\frac{P_S \cdot L_S}{A_S \cdot E_S} = \frac{P_C \cdot L_C}{A_C \cdot E_C}
\]

\[\Rightarrow \quad \frac{P_S \times 1150}{(6 \times 200) \times E_S} = \frac{P_C \times 1150}{\left(\frac{\pi \times 300^2}{4} - (6 \times 200)\right) \times E_C}\]

\[1370 \text{ kN}\]
**Solution:**

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\frac{P_S \cdot L_S}{A_S \cdot E_S} = \frac{P_C \cdot L_C}{A_C \cdot E_C}
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\[
\Rightarrow \frac{P_S \times 1150}{1200 \times (200 \times 10^3)} = \frac{P_C \times 1150}{\left(\frac{\pi \times 300^2}{4} - 1200\right) \times (25 \times 10^3)}
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\[
1370 \text{ kN}
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**Problems Involving Two Materials**

**Solution:**

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\frac{P_S \cdot L_S}{A_S \cdot E_S} = \frac{P_C \cdot L_C}{A_C \cdot E_C}
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\[\Rightarrow \frac{P_S}{1200 \times 200} = \frac{P_C}{69486 \times 25}
\]

1370 kN
Problems Involving Two Materials

Solution:

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1370 kN

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Solution:

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\frac{P_S \cdot L_S}{A_S \cdot E_S} = \frac{P_C \cdot L_C}{A_C \cdot E_C}
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\Rightarrow P_S = \frac{1200 \times 200 \cdot P_C}{69486 \times 25}
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\Rightarrow P_S = 0.13816P_C
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1370 kN
**Problems Involving Two Materials**

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\frac{P_S \cdot L_S}{A_S \cdot E_S} = \frac{P_C \cdot L_C}{A_C \cdot E_C}
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\[\Rightarrow \quad P_S = \frac{1200 \times 200}{69486 \times 25} \cdot P_C
\]

\[\Rightarrow \quad P_S = 0.13816P_C
\]

We now have two equations for the two unknowns, \(P_S\) and \(P_C\).
Solution:

\[ P_S + P_C = 1370 \]

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\[ \Rightarrow P_C = \frac{1370}{1+0.13816} \]

1370 kN
Problems Involving Two Materials

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\[ \Rightarrow P_C = 1203.7 \text{ kN} \]
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\[ \Rightarrow P_C = 1203.7 \text{ kN} \]
\[ \Rightarrow P_S = 166.3 \text{ kN} \]
**Solution:**

\[ P_S + P_C = 1370 \]
\[ P_S = 0.13816P_C \]
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\[ \Rightarrow P_C = 1203.7 \text{ kN} \]
\[ \Rightarrow P_S = 166.3 \text{ kN} \]

We can now calculate the stress in the steel and in the concrete.
**Solution:** Find the stress in the concrete:
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P_C = 1098 \text{ kN}
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\[ P_C = 1098 \text{ kN} \]

\[ \Rightarrow \sigma_C = \frac{P_C}{A} \]

\[ \Rightarrow \sigma_C = \frac{1098}{\frac{\pi \times 300^2}{4} - (6 \times 200)} \]

\[ \Rightarrow \sigma_C = 0.0580 \frac{\text{kN}}{\text{mm}^2} \]
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\[ \Rightarrow \sigma_C = 58.0 \text{ MPa} \]
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\[ P_C = 1098 \text{ kN} \]

\[ \Rightarrow \sigma_C = \frac{P_C}{A} \]

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\[ \Rightarrow \sigma_C = 0.0580 \frac{\text{kN}}{\text{mm}^2} \]

\[ \Rightarrow \sigma_C = 58.0 \text{ MPa} \]

Find the stress in the steel:
**Problems Involving Two Materials**

**Solution:** Find the stress in the concrete:

\[ P_C = 1098 \text{ kN} \]

\[ \Rightarrow \sigma_C = \frac{P_C}{A} \]

\[ \Rightarrow \sigma_C = \frac{1098}{\frac{\pi \times 300^2}{4} - (6 \times 200)} \]

\[ \Rightarrow \sigma_C = 0.0580 \frac{\text{kN}}{\text{mm}^2} \]

\[ \Rightarrow \sigma_C = 58.0 \text{ MPa} \]

Find the stress in the steel:

\[ P_S = 152 \text{ kN} \]
**Solution:** Find the stress in the concrete:

\[
P_C = 1098 \text{ kN}
\]

\[
\Rightarrow \sigma_C = \frac{P_C}{A}
\]

\[
\Rightarrow \sigma_C = \frac{1098}{\frac{\pi \times 300^2}{4} - (6 \times 200)}
\]

\[
\Rightarrow \sigma_C = 0.0580 \frac{\text{kN}}{\text{mm}^2}
\]

\[
\Rightarrow \sigma_C = 58.0 \text{ MPa}
\]

Find the stress in the steel:

\[
P_S = 152 \text{ kN}
\]

\[
\Rightarrow \sigma_S = \frac{152}{(6 \times 200)}
\]
**Problems Involving Two Materials**

**Solution:** Find the stress in the concrete:

\[ P_C = 1098 \text{ kN} \]

\[ \Rightarrow \sigma_C = \frac{P_C}{A} \]

\[ \Rightarrow \sigma_C = \frac{1098}{\frac{\pi \times 300^2}{4} - (6 \times 200)} \]

\[ \Rightarrow \sigma_C = 0.0580 \frac{\text{kN}}{\text{mm}^2} \]

\[ \Rightarrow \sigma_C = 58.0 \text{ MPa} \]

Find the stress in the steel:

\[ P_S = 152 \text{ kN} \]

\[ \Rightarrow \sigma_S = \frac{152}{(6 \times 200)} \]

\[ \Rightarrow \sigma_S = 0.1267 \frac{\text{kN}}{\text{mm}^2} \]
**Solution**: Find the stress in the concrete:

\[
PC = 1098 \text{ kN}
\]

\[
\Rightarrow \sigma_C = \frac{PC}{A}
\]

\[
\Rightarrow \sigma_C = \frac{1098}{\frac{\pi \times 300^2}{4} - (6 \times 200)}
\]

\[
\Rightarrow \sigma_C = 0.0580 \frac{\text{kn}}{\text{mm}^2}
\]

\[
\Rightarrow \sigma_C = 58.0 \text{ MPa}
\]

Find the stress in the steel:

\[
PS = 152 \text{ kN}
\]

\[
\Rightarrow \sigma_S = \frac{152}{6 \times 200}
\]

\[
\Rightarrow \sigma_S = 0.1267 \frac{\text{kn}}{\text{mm}^2}
\]

\[
\Rightarrow \sigma_S = 126.7 \text{ MPa}
\]
Problems Involving Two Materials

Solution: Find the deformation in the concrete:
Solution: Find the deformation in the concrete:

$$\delta_C = \frac{P_C \cdot L_C}{A_C \cdot E_C}$$
**Solution:** Find the deformation in the concrete:

\[
\delta_C = \frac{P_C \cdot L_C}{A_C \cdot E_C} \Rightarrow \delta_C = \frac{1098 \times (3.5 \times 10^3)}{(\frac{\pi \times 300^2}{4} - 1200) \times (25 \times 10^3)}
\]
Solution: Find the deformation in the concrete:

\[
\delta_C = \frac{P_C \cdot L_C}{A_C \cdot E_C}
\]

\[
\Rightarrow \delta_C = \frac{1098 \times (3.5 \times 10^3)}{(\pi \times 300^2/4 - 1200) \times (25 \times 10^3)}
\]

\[
\Rightarrow \delta_C = 0.00221 \text{ mm}
\]
Solution: Find the deformation in the concrete:

\[ \delta_C = \frac{P_C \cdot L_C}{A_C \cdot E_C} \]

\[ \Rightarrow \delta_C = \frac{1098 \times (3.5 \times 10^3)}{\left(\frac{\pi \times 300^2}{4} - 1200\right) \times (25 \times 10^3)} \]

\[ \Rightarrow \delta_C = 0.00221 \text{ mm} \]

Find the deformation in the steel (if we’ve done our calculations correctly, then \( \delta_S = \delta_C \)):

\[ \tilde{\delta}_S = \frac{P_S \cdot L_S}{A_S \cdot E_S} \]
Solution: Find the deformation in the concrete:

$$\delta_C = \frac{P_C \cdot L_C}{A_C \cdot E_C}$$

$$\Rightarrow \delta_C = \frac{1098 \times (3.5 \times 10^3)}{(\frac{\pi \times 300^2}{4} - 1200) \times (25 \times 10^3)}$$

$$\Rightarrow \delta_C = 0.00221 \text{ mm}$$

Find the deformation in the steel (if we’ve done our calculations correctly, then $\delta_S = \delta_C$):

$$\delta_S = \frac{P_S \cdot L_S}{A_S \cdot E_S}$$

$$\Rightarrow \delta_S = \frac{152 \times (3.5 \times 10^3)}{1200 \times (200 \times 10^3)}$$
**Solution:** Find the deformation in the concrete:

\[
\delta_C = \frac{P_C \cdot L_C}{A_C \cdot E_C} \times \left( \frac{3.5 \times 10^3}{\pi \times 300^2 / 4 - 1200} \times (25 \times 10^3) \right)
\]

\[
\Rightarrow \delta_C = 0.00221 \text{ mm}
\]

Find the deformation in the steel (if we’ve done our calculations correctly, then \(\delta_S = \delta_C\)):

\[
\delta_S = \frac{P_S \cdot L_S}{A_S \cdot E_S} \times \left( \frac{3.5 \times 10^3}{1200 \times (200 \times 10^3)} \right)
\]

\[
\Rightarrow \delta_S = 0.00222 \text{ mm}
\]
Solution: Find the deformation in the concrete:

\[ \delta_C = \frac{P_C \cdot L_C}{A_C \cdot E_C} \]

\[ \Rightarrow \delta_C = \frac{1098 \times (3.5 \times 10^3)}{\left(\frac{\pi \times 300^2}{4} - 1200\right) \times (25 \times 10^3)} \]

\[ \Rightarrow \delta_C = 0.00221 \text{ mm} \]

Find the deformation in the steel (if we’ve done our calculations correctly, then \( \delta_S = \delta_C \)):

\[ \delta_S = \frac{P_S \cdot L_S}{A_S \cdot E_S} \]

\[ \Rightarrow \delta_S = \frac{152 \times (3.5 \times 10^3)}{1200 \times (200 \times 10^3)} \]

\[ \Rightarrow \delta_S = 0.00222 \text{ mm} \]

The small difference in deformation is due to rounding errors.
**Exercise:** A hollow square steel structural section has outside dimensions of 115 mm × 115 mm and inside dimensions of 105 mm × 105 mm. It is filled with concrete, as shown in plan view (upper right). The section is 3.5 m and supports a compressive load of 250 kN.

\( E_S = 200 \text{ GPa} \) and \( E_C = 20 \text{ GPa} \).

Find \( \sigma_S, \sigma_C \) and \( \delta \).
**Solution:** Find the areas of the steel and of the concrete:
**Solution:** Find the areas of the steel and of the concrete:

\[ A_S = (2 \times 115 \times 5) + (2 \times 105 \times 5) \]
**Solution:** Find the areas of the steel and of the concrete:

\[
A_S = (2 \times 115 \times 5) + (2 \times 105 \times 5)
\]

\[
= 2200 \text{ mm}^2
\]
**Solution:** Find the areas of the steel and of the concrete:

\[ \begin{align*}
A_S &= (2 \times 115 \times 5) + (2 \times 105 \times 5) \\
    &= 2200 \text{ mm}^2 \\
A_C &= 105 \times 105
\end{align*} \]
**Solution:** Find the areas of the steel and of the concrete:

\[
A_S = (2 \times 115 \times 5) + (2 \times 105 \times 5) \\
= 2200 \text{ mm}^2
\]

\[
A_C = 105 \times 105 \\
= 11025 \text{ mm}^2
\]
**Solution:** Find the areas of the steel and of the concrete:

\[
A_S = (2 \times 115 \times 5) + (2 \times 105 \times 5) \\
= 2200 \text{ mm}^2
\]

\[
A_C = 105 \times 105 \\
= 11025 \text{ mm}^2
\]

Let \( P_S \) be the reaction force of the steel and \( P_C \) the reaction force of the concrete.
Solution: Find the areas of the steel and of the concrete:

\[
A_S = (2 \times 115 \times 5) + (2 \times 105 \times 5) = 2200 \text{ mm}^2
\]

\[
A_C = 105 \times 105 = 11025 \text{ mm}^2
\]

Let \( P_S \) be the reaction force of the steel and \( P_C \) the reaction force of the concrete. Then,

\[
\Sigma F_y = P_S + P_C - 250 = 0
\]
Solution: Find the areas of the steel and of the concrete:

\[ A_S = (2 \times 115 \times 5) + (2 \times 105 \times 5) \]
\[ = 2200 \text{ mm}^2 \]

\[ A_C = 105 \times 105 \]
\[ = 11025 \text{ mm}^2 \]

Let \( P_S \) be the reaction force of the steel and \( P_C \) the reaction force of the concrete. Then,

\[ \Sigma F_y = P_S + P_C - 250 = 0 \]
\[ P_S + P_C = 250 \text{ kN} \]
**Solution:** The steel casing and the concrete both deform by the same amount.
Solution: The steel casing and the concrete both deform by the same amount

\[ \frac{P_C \cdot L_C}{A_C \cdot E_C} = \frac{P_S \cdot L_S}{A_S \cdot E_S} \]
**Solution:** The steel casing and the concrete both deform by the same amount

\[
\frac{P_C \cdot L_C}{A_C \cdot E_C} = \frac{P_S \cdot L_S}{A_S \cdot E_S}
\]

\[
\Rightarrow \frac{(P_C \times 10^3) \times 175}{11025 \times (200 \times 10^3)} = \frac{(P_S \times 10^3) \times 175}{2200 \times (200 \times 10^3)}
\]
**Solution:** The steel casing and the concrete both deform by the same amount

\[
\frac{P_C \cdot L_C}{A_C \cdot E_C} = \frac{P_S \cdot L_S}{A_S \cdot E_S}
\]

\[
\Rightarrow \frac{(P_C \times 10^3) \times 175}{11025 \times (200 \times 10^3)} = \frac{(P_S \times 10^3) \times 175}{2200 \times (200 \times 10^3)}
\]

\[
\Rightarrow \frac{P_C}{11025 \times 20} = \frac{P_S}{2200 \times 200}
\]
**Solution:** The steel casing and the concrete both deform by the same amount

\[
\frac{P_C \cdot L_C}{A_C \cdot E_C} = \frac{P_S \cdot L_S}{A_S \cdot E_S}
\]

\[
\Rightarrow \frac{(P_C \times 10^3) \times 175}{11025 \times (200 \times 10^3)} = \frac{(P_S \times 10^3) \times 175}{2200 \times (200 \times 10^3)}
\]

\[
\Rightarrow \frac{P_C}{11025 \times 20} = \frac{P_S}{2200 \times 200}
\]

\[
\Rightarrow P_C = \frac{11025 \times 20}{2200 \times 200} \cdot P_S
\]
**Solution:** The steel casing and the concrete both deform by the same amount

\[
\frac{P_C \cdot L_C}{A_C \cdot E_C} = \frac{P_S \cdot L_S}{A_S \cdot E_S}
\]

\[
\Rightarrow \frac{(P_C \times 10^3) \times 175}{11025 \times (200 \times 10^3)} = \frac{(P_S \times 10^3) \times 175}{2200 \times (200 \times 10^3)}
\]

\[
\Rightarrow \frac{P_C}{11025 \times 20} = \frac{P_S}{2200 \times 200}
\]

\[
\Rightarrow P_C = \frac{11025 \times 20}{2200 \times 200} \cdot P_S
\]

\[
\Rightarrow P_C = 0.5011P_S
\]
**Solution:** The steel casing and the concrete both deform by the same amount

\[
\frac{P_C \cdot L_C}{A_C \cdot E_C} = \frac{P_S \cdot L_S}{A_S \cdot E_S}
\]

\[
\Rightarrow \frac{(P_C \times 10^3) \times 175}{11025 \times (200 \times 10^3)} = \frac{(P_S \times 10^3) \times 175}{2200 \times (200 \times 10^3)}
\]

\[
\Rightarrow \frac{P_C}{11025 \times 20} = \frac{P_S}{2200 \times 200}
\]

\[
\Rightarrow P_C = \frac{11025 \times 20}{2200 \times 200} \cdot P_S
\]

\[
\Rightarrow P_C = 0.5011P_S
\]

\[\sum F_y = 0 \text{ so} \]

\[P_C + P_S - 250 = 0\]
**Solution:** The steel casing and the concrete both deform by the same amount

\[ \frac{P_C \cdot L_C}{A_C \cdot E_C} = \frac{P_S \cdot L_S}{A_S \cdot E_S} \]

\[ \Rightarrow \frac{(P_C \times 10^3) \times 175}{11025 \times (200 \times 10^3)} = \frac{(P_S \times 10^3) \times 175}{2200 \times (200 \times 10^3)} \]

\[ \Rightarrow \frac{P_C}{11025 \times 20} = \frac{P_S}{2200 \times 200} \]

\[ \Rightarrow P_C = \frac{11025 \times 20}{2200 \times 200} \cdot P_S \]

\[ \Rightarrow P_C = 0.5011 P_S \]

\[ \Sigma F_y = 0 \text{ so} \]

\[ P_C + P_S - 250 = 0 \]

\[ \Rightarrow P_C = 250 - P_S \]
**Solution:** The steel casing and the concrete both deform by the same amount

\[
\frac{P_C \cdot L_C}{A_C \cdot E_C} = \frac{P_S \cdot L_S}{A_S \cdot E_S}
\]

\[
\Rightarrow \frac{(P_C \times 10^3) \times 175}{11025 \times (200 \times 10^3)} = \frac{(P_S \times 10^3) \times 175}{2200 \times (200 \times 10^3)}
\]

\[
\Rightarrow \frac{P_C}{11025 \times 20} = \frac{P_S}{2200 \times 200}
\]

\[
\Rightarrow P_C = \frac{11025 \times 20}{2200 \times 200} \cdot P_S
\]

\[
\Rightarrow P_C = 0.5011P_S
\]

\[
\Sigma F_y = 0 \text{ so }
\]

\[
P_C + P_S - 250 = 0
\]

\[
\Rightarrow P_C = 250 - P_S
\]

Now we have two equations for the two unknowns, \(P_C\) and \(P_S\)
Solution:

\[ P_C = 0.5011P_S \]

\[ P_C = 250 - P_S \]
Solution:

\[ P_C = 0.5011P_S \]

\[ P_C = 250 - P_S \]

\[ \Rightarrow 250 - P_S = 0.5011P_S \]
**Solution:**

\[
P_C = 0.5011P_S
\]

\[
P_C = 250 - P_S
\]

\[
\Rightarrow 250 - P_S = 0.5011P_S
\]

\[
\Rightarrow P_S = \frac{250}{1 + 0.5011}
\]
Solution:

\[ P_C = 0.5011P_S \]

\[ P_C = 250 - P_S \]

\[ \Rightarrow 250 - P_S = 0.5011P_S \]

\[ \Rightarrow P_S = \frac{250}{1+0.5011} \]

\[ \Rightarrow P_S = 166.5 \text{ kN} \]
Solution:

\[ PC = 0.5011PS \]
\[ PC = 250 - PS \]
\[ \Rightarrow 250 - PS = 0.5011PS \]
\[ \Rightarrow PS = \frac{250}{1 + 0.5011} \]
\[ \Rightarrow PS = 166.5 \text{ kN} \]
\[ \Rightarrow PC = 83.5 \text{ kN} \]
Solution:

\[ P_C = 0.5011P_S \]
\[ P_C = 250 - P_S \]
\[ 250 - P_S = 0.5011P_S \]
\[ P_S = \frac{250}{1+0.5011} \]
\[ P_S = 166.5 \text{ kN} \]
\[ P_C = 83.5 \text{ kN} \]

Now, find the stress in the steel:
Solution:

\[ P_C = 0.5011P_S \]

\[ P_C = 250 - P_S \]

\[ \Rightarrow 250 - P_S = 0.5011P_S \]

\[ \Rightarrow P_S = \frac{250}{1+0.5011} \]

\[ \Rightarrow P_S = 166.5 \text{ kN} \]

\[ \Rightarrow P_C = 83.5 \text{ kN} \]

Now, find the stress in the steel:

\[ \sigma_S = \frac{P_S}{A_S} \]
Solution:

\[ PC = 0.5011 PS \]
\[ PC = 250 - PS \]
\[ \Rightarrow 250 - PS = 0.5011PS \]
\[ \Rightarrow PS = \frac{250}{1+0.5011} \]
\[ \Rightarrow PS = 166.5 \text{ kN} \]
\[ \Rightarrow PC = 83.5 \text{ kN} \]

Now, find the stress in the steel:

\[ \sigma_S = \frac{PS}{A_S} \]
\[ \Rightarrow \sigma_S = \frac{166.5 \times 10^3}{2200} \]
Solution:

\[ P_C = 0.5011P_S \]

\[ P_C = 250 - P_S \]

\[ 250 - P_S = 0.5011P_S \]

\[ P_S = \frac{250}{1 + 0.5011} \]

\[ PS = 166.5 \text{ kN} \]

\[ PC = 83.5 \text{ kN} \]

Now, find the stress in the steel:

\[ \sigma_S = \frac{P_S}{A_S} \]

\[ \sigma_S = \frac{166.5 \times 10^3}{2200} \]

\[ \sigma_S = 75.7 \text{ MPa} \]
Solution: Now, find the stress in the concrete:
Solution: Now, find the stress in the concrete:

\[ \sigma_C = \frac{P_C}{A_C} \]
**Solution:** Now, find the stress in the concrete:

\[
\sigma_C = \frac{P_C}{A_C}
\]

\[
\Rightarrow \sigma_C = \frac{83.5 \times 10^3}{11025}
\]
**Solution:** Now, find the stress in the concrete:

\[
\sigma_C = \frac{P_C}{A_C}
\]

\[
\Rightarrow \sigma_C = \frac{83.5 \times 10^3}{11025}
\]

\[
\Rightarrow \sigma_C = 7.57 \text{ MPa}
\]
**Solution:** Now, find the stress in the concrete:

\[ \sigma_C = \frac{P_C}{A_C} \]

\[ \Rightarrow \sigma_C = \frac{83.5 \times 10^3}{11025} \]

\[ \Rightarrow \sigma_C = 7.57 \text{ MPa} \]

Now, find the deformation:
**Solution:** Now, find the stress in the concrete:

\[
\sigma_C = \frac{P_C}{A_C}
\]

\[
\Rightarrow \sigma_C = \frac{83.5 \times 10^3}{11025}
\]

\[
\Rightarrow \sigma_C = 7.57 \text{ MPa}
\]

Now, find the deformation:

\[
\delta = \frac{P_C \cdot L_C}{A_C \cdot E_C}
\]
**Solution:** Now, find the stress in the concrete:

\[ \sigma_C = \frac{P_C}{A_C} \]

\[ \Rightarrow \sigma_C = \frac{8.35 \times 10^3}{11025} \]

\[ \Rightarrow \sigma_C = 7.57 \text{ MPa} \]

Now, find the deformation:

\[ \delta = \frac{P_C \cdot L_C}{A_C \cdot E_C} \]

\[ \Rightarrow \delta = \frac{(8.35 \times 10^3) \times 175}{11025 \times (20 \times 10^3)} \]
Solution: Now, find the stress in the concrete:

\[ \sigma_C = \frac{P_C}{A_C} \]

\[ \Rightarrow \sigma_C = \frac{83.5 \times 10^3}{11025} \]

\[ \Rightarrow \sigma_C = 7.57 \text{ MPa} \]

Now, find the deformation:

\[ \delta = \frac{P_C \cdot L_C}{A_C \cdot E_C} \]

\[ \Rightarrow \delta = \frac{(83.5 \times 10^3) \times 175}{11025 \times (20 \times 10^3)} \]

\[ \Rightarrow \delta = 0.0663 \text{ mm} \]